

Five-loop $\sqrt{\epsilon}$ -expansions for random Ising model and marginal spin dimensionality for cubic systems.

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The $\sqrt{\epsilon}$ -expansions for critical exponents of the weakly-disordered Ising model are calculated up to the five-loop order and found to possess coefficients with irregular signs and values. The estimate $n_c = 2.855$ for the marginal spin dimensionality of the cubic model is obtained by the Pade-Borel resummation of corresponding five-loop ϵ -expansion.

Keywords: Impure Ising model; $\sqrt{\epsilon}$ -expansion; Five-loop approximation;
Cubic systems; Marginal spin dimensionality

I. INTRODUCTION

The critical behaviour of weakly-disordered quenched systems undergoing continuous phase transitions is of great interest. The study of critical properties of random spin systems in which the local energy density couples to quenched disorder has a long history going back to the classical papers by A. B. Harris and T. C. Lubensky [1,2] and D. E. Khmel'nitskii [3]. They initiated a considerable progress in studying disordered systems by applying the conventional field-theoretical renormalization-group (RG) approach based on the standard scalar φ^4 theory with n -component order parameter in $(4 - \epsilon)$ space dimensions. At present it is commonly believed that the RG approach provides a thorough understanding how weak disorder affects thermodynamic properties of random systems in a close vicinity of the Curie point.

According to the Harris criterion, critical exponents of weakly disordered Ising model should differ from those for the pure system [4]. The first regular method for calculating their values, famous $\sqrt{\epsilon}$ -expansion, was invented more than 20 years ago [1,2,3] but turned out to be numerically ineffective, at least in lower orders in $\sqrt{\epsilon}$. Fair numerical estimates for critical exponents of the random Ising model (RIM) were obtained in the framework of the renormalization-group approach in three dimensions from two-loop [5], three-loop [6] and four-loop [7,8] RG expansions. The RG method at fixed dimensions proved also to be efficient when used to calculate critical exponents of the two-dimensional RIM and the marginal value n_c of the order parameter dimensionality n for the cubic model [8,9,10,11]; as is known, n_c separates the region where the system becomes effectively isotropic approaching the critical point ($n < n_c$) from that of the essentially anisotropic critical behaviour ($n > n_c$).

Recently, H. Kleinert and V. Schulte-Frohlinde have found the RG functions for the $(4 - \epsilon)$ -dimensional hypercubic model in the five-loop approximation [12]. To obtain the RG series of unprecedented length for the model with two quartic coupling constants these

authors have employed the early results of five-loop RG calculations for $O(n)$ -symmetric φ^4 field theory [13]. It is well known that in the replica limit $n \rightarrow 0$ the scalar φ^4 field theory with $O(n)$ -symmetric and hypercubic self-interactions describes the critical behaviour of the RIM provided the coupling constants have proper signs [14]. Hence, the RG expansions obtained in [12] may be used for calculation of RIM critical exponents as power series in $\sqrt{\epsilon}$ or, more precisely, for extension of known three-loop expansions [15,16] up to five-loop (ϵ^2) terms. To find such five-loop expansions is the main goal of this paper.

Another goal is to get numerical estimate for n_c starting from the ϵ -expansion for this quantity obtained in Ref. [12] and to compare this estimate with its analogs found earlier from RG expansions in three dimensions [7,9,10]. If the results given by these two approaches are in accord we shall be able, at last, to answer the old question: is a cubic crystal effectively “isotropic” at the critical point?

II. RG FUNCTIONS AND CRITICAL EXPONENTS FOR RANDOM ISING MODEL

We begin with the standard Landau Hamiltonian for a model with hypercubic anisotropy, describing numerous magnetic and structural phase transitions in solids. It reads:

$$H = \int d^d x \left[\frac{1}{2} (\partial_\mu \varphi)^2 + \frac{1}{2} m_o^2 \varphi^2 + \frac{1}{4!} u_o (\varphi^2)^2 + \frac{1}{4!} v_o \sum_{a=1}^n \varphi_a^4 \right] ,$$

$$\varphi^2 = \sum_{a=1}^n \varphi_a^2 , \quad (\partial_\mu \varphi)^2 = \sum_{a=1}^n (\partial_\mu \varphi_a)^2 , \quad (2.1)$$

where φ is an n -component order parameter, $m_o^2 \sim \tau$, with $\tau = \frac{T-T_c}{T_c}$ being the reduced deviation from the mean-field transition temperature. In the replica limit, the Hamiltonian Eq. (2.1) is known to describe the RIM provided $v_o > 0$ and $u_o < 0$.

If we set n to zero in formulas obtained in [12], we arrive, directly, at five-loop β -functions and critical exponents for the model under consideration:

$$\begin{aligned}
\frac{\beta_u(u, v)}{u} = & -\epsilon + \frac{8}{3}u + 2v - \frac{14}{3}u^2 - \frac{22}{3}uv - \frac{5}{3}v^2 + u^3\left(\frac{370}{27} + \frac{88}{9}\zeta(3)\right) + u^2v\left(\frac{659}{18} + \frac{64}{3}\zeta(3)\right) \\
& + uv^2\left(\frac{107}{4} + 8\zeta(3)\right) + 7v^3 + u^4\left(-\frac{24581}{486} - \frac{4664}{81}\zeta(3) + \frac{352}{27}\zeta(4) - \frac{2480}{27}\zeta(5)\right) \\
& + u^3v\left(-\frac{15967}{81} - \frac{4856}{27}\zeta(3) + \frac{340}{9}\zeta(4) - \frac{2560}{9}\zeta(5)\right) + u^2v^2\left(-\frac{13433}{54} - \frac{1456}{9}\zeta(3) \right. \\
& + \left.\frac{64}{3}\zeta(4) - \frac{2000}{9}\zeta(5)\right) + uv^3\left(-\frac{4867}{36} - 50\zeta(3) - 8\zeta(4) - \frac{160}{3}\zeta(5)\right) \\
& + v^4\left(-\frac{477}{16} - 3\zeta(3) - 6\zeta(4)\right) + u^5\left(\frac{17158}{81} + \frac{27382}{81}\zeta(3) + \frac{1088}{27}\zeta(3)^2 - \frac{880}{9}\zeta(4) \right. \\
& + \left.\frac{55028}{81}\zeta(5) - \frac{6200}{27}\zeta(6) + \frac{25774}{27}\zeta(7)\right) + u^4v\left(\frac{537437}{486} + \frac{116759}{81}\zeta(3) + \frac{3148}{27}\zeta(3)^2 \right. \\
& - \left.\frac{10177}{27}\zeta(4) + \frac{75236}{27}\zeta(5) - \frac{24050}{27}\zeta(6) + \frac{11564}{3}\zeta(7)\right) + u^3v^2\left(\frac{1314497}{648} \right. \\
& + \left.\frac{171533}{81}\zeta(3) + \frac{1384}{27}\zeta(3)^2 - \frac{23105}{54}\zeta(4) + \frac{96794}{27}\zeta(5) - \frac{25400}{27}\zeta(6) + \frac{14210}{3}\zeta(7)\right) \\
& + u^2v^3\left(\frac{2281727}{1296} + \frac{37789}{27}\zeta(3) - \frac{544}{9}\zeta(3)^2 - \frac{337}{3}\zeta(4) + \frac{17444}{9}\zeta(5) - \frac{1600}{9}\zeta(6) \right. \\
& + \left.2352\zeta(7)\right) + uv^4\left(\frac{1336801}{1728} + \frac{5495}{12}\zeta(3) - \frac{190}{3}\zeta(3)^2 + \frac{141}{2}\zeta(4) \right. \\
& + \left.\frac{1145}{3}\zeta(5) + \frac{575}{3}\zeta(6) + 441\zeta(7)\right) + v^5\left(\frac{158849}{1152} + \frac{1519}{24}\zeta(3) \right. \\
& - \left.18\zeta(3)^2 + \frac{65}{2}\zeta(4) + 2\zeta(5) + 75\zeta(6)\right) , \tag{2.2}
\end{aligned}$$

$$\begin{aligned}
\frac{\beta_v(u, v)}{v} = & -\epsilon + 3v + 4u - \frac{17}{3}v^2 - \frac{46}{3}uv - \frac{82}{9}u^2 + v^3\left(\frac{145}{8} + 12\zeta(3)\right) + uv^2\left(\frac{131}{2} + 48\zeta(3)\right) \\
& + u^2v\left(\frac{325}{4} + 64\zeta(3)\right) + u^3\left(\frac{821}{27} + \frac{224}{9}\zeta(3)\right) + v^4\left(-\frac{3499}{48} - 78\zeta(3) + 18\zeta(4) \right. \\
& - \left.120\zeta(5)\right) + uv^3\left(-\frac{1004}{3} - 387\zeta(3) + 96\zeta(4) - 600\zeta(5)\right) + u^2v^2\left(-\frac{10661}{18} \right. \\
& - \left.724\zeta(3) + 184\zeta(4) - \frac{3440}{3}\zeta(5)\right) + u^3v\left(-\frac{12349}{27} - \frac{5312}{9}\zeta(3) \right. \\
& + \left.\frac{440}{3}\zeta(4) - 960\zeta(5)\right) + u^4\left(-\frac{19679}{162} - 168\zeta(3) + 40\zeta(4) - \frac{7280}{27}\zeta(5)\right) \\
& + v^5\left(\frac{764621}{2304} + \frac{7965}{16}\zeta(3) + 45\zeta(3)^2 - \frac{1189}{8}\zeta(4) + 987\zeta(5) \right. \\
& - \left.\frac{675}{2}\zeta(6) + 1323\zeta(7)\right) + uv^4\left(\frac{1067507}{576} + \frac{35083}{12}\zeta(3) + 288\zeta(3)^2 \right. \\
& - \left.\frac{3697}{4}\zeta(4) + 5920\zeta(5) - 2100\zeta(6) + 7938\zeta(7)\right) \\
& + u^2v^3\left(\frac{3633377}{864} + \frac{125459}{18}\zeta(3) + \frac{2266}{3}\zeta(3)^2 - 2263\zeta(4) \right. \\
& + \left.14328\zeta(5) - \frac{15575}{3}\zeta(6) + 19404\zeta(7)\right) + u^3v^2\left(\frac{9309907}{1944} + \frac{224804}{27}\zeta(3) \right.
\end{aligned}$$

$$\begin{aligned}
& + \frac{3032}{3}\zeta(3)^2 - \frac{73018}{27}\zeta(4) + \frac{155692}{9}\zeta(5) - 6300\zeta(6) + 23912\zeta(7) \Big) \\
& + u^4 v \left(\frac{1279979}{486} + \frac{784621}{162}\zeta(3) + \frac{18154}{27}\zeta(3)^2 - \frac{83837}{54}\zeta(4) + \frac{275510}{27}\zeta(5) \right. \\
& - \frac{98975}{27}\zeta(6) + \frac{43120}{3}\zeta(7) \Big) + u^5 \left(\frac{389095}{729} + \frac{259358}{243}\zeta(3) + \frac{13288}{81}\zeta(3)^2 \right. \\
& - \frac{27166}{81}\zeta(4) + \frac{179696}{81}\zeta(5) - \frac{63500}{81}\zeta(6) + \frac{28420}{9}\zeta(7) \Big) , \tag{2.3}
\end{aligned}$$

$$\begin{aligned}
\eta(u, v) = & \frac{1}{9}u^2 + \frac{1}{3}uv + \frac{1}{6}v^2 - \frac{2}{27}u^3 - \frac{1}{3}u^2v - \frac{3}{8}uv^2 - \frac{1}{8}v^3 + u^4 \frac{125}{324} \\
& + u^3v \frac{125}{54} + u^2v^2 \frac{145}{36} + \frac{65}{24}uv^3 + v^4 \frac{65}{96} + u^5 \left(-\frac{1204}{729} + \frac{46}{243}\zeta(3) - \frac{44}{81}\zeta(4) \right) \\
& + u^4v \left(-\frac{3010}{243} + \frac{115}{81}\zeta(3) - \frac{110}{27}\zeta(4) \right) + u^3v^2 \left(-\frac{58177}{1944} + \frac{191}{54}\zeta(3) - \frac{260}{27}\zeta(4) \right) \\
& + u^2v^3 \left(-\frac{13741}{432} + \frac{67}{18}\zeta(3) - 10\zeta(4) \right) + uv^4 \left(-\frac{18545}{1152} + \frac{15}{8}\zeta(3) - 5\zeta(4) \right) \\
& + v^5 \left(-\frac{3709}{1152} + \frac{3}{8}\zeta(3) - \zeta(4) \right) , \tag{2.4}
\end{aligned}$$

$$\begin{aligned}
\nu(u, v)^{-1} = & 2 - \frac{2}{3}u - v + \frac{5}{9}u^2 + \frac{5}{3}uv + \frac{5}{6}v^2 - \frac{37}{18}u^3 - \frac{37}{4}u^2v - \frac{251}{24}uv^2 \\
& - \frac{7}{2}v^3 - u^4 \left(-\frac{7765}{972} - \frac{68}{81}\zeta(3) - \frac{44}{27}\zeta(4) \right) - u^3v \left(-\frac{7765}{162} \right. \\
& - \frac{136}{27}\zeta(3) - \frac{88}{9}\zeta(4) \Big) - u^2v^2 \left(-\frac{9199}{108} - \frac{26}{3}\zeta(3) - \frac{52}{3}\zeta(4) \right) \\
& - uv^3 \left(-\frac{4243}{72} - \frac{19}{3}\zeta(3) - 12\zeta(4) \right) - v^4 \left(-\frac{477}{32} - \frac{3}{2}\zeta(3) - 3\zeta(4) \right) , \tag{2.5}
\end{aligned}$$

where numbers $\zeta(3) = 1.202056903$; $\zeta(4) = 1.082323234$; $\zeta(5) = 1.036927755$; $\zeta(6) = 1.017343062$; $\zeta(7) = 1.008349277$ are the values of the Riemann ζ -function. The coordinates of the random infrared-free fixed point being zeros of the above β -functions, they are found in the following form [15,16]:

$$u^* = -A\sqrt{\epsilon} + B\epsilon + C\sqrt{\epsilon}^3 + D\epsilon^2 + K\sqrt{\epsilon}^5 + \dots , \tag{2.6}$$

$$v^* = \frac{4}{3}A\sqrt{\epsilon} + (F - \frac{4}{3}B)\epsilon + (G - \frac{4}{3}C)\sqrt{\epsilon}^3 + (H - \frac{4}{3}D)\epsilon^2 + (L - \frac{4}{3}K)\sqrt{\epsilon}^5 + \dots . \tag{2.7}$$

Substituting u^* and v^* into the beta-functions, we get algebraic equations which decouple into 4 pairs of equations for A and F, B and G, C and H, and, at last, for D and L, respectively.

Note, that the coefficient K may be computed only in the sixth-loop approximation. After straightforward but cumbersome calculations one is led to the following expressions:

$$\begin{aligned}
u^* = & -\sqrt{\epsilon} \frac{3\sqrt{318}}{106} + \epsilon \left(\frac{567\zeta(3)}{2809} + \frac{990}{2809} \right) - \sqrt{\epsilon^3} \frac{9\sqrt{318}}{252495392} \left(317520\zeta(3)^2 + 775536\zeta(3) \right. \\
& + 5(114365 - 137376\zeta(5)) \Big) + \epsilon^2 \frac{9}{3345563944} \left(96018048\zeta(3)^3 + 326732616\zeta(3)^2 \right. \\
& + 6\zeta(3)(26149279 - 64910160\zeta(5)) + 29477646\zeta(4) \\
& \left. - 309476010\zeta(5) - 367914393\zeta(7) + 101727760 \right) , \tag{2.8}
\end{aligned}$$

$$\begin{aligned}
v^* = & \sqrt{\epsilon} \frac{2\sqrt{318}}{53} - 36\epsilon \frac{(21\zeta(3) + 19)}{2809} + \sqrt{\epsilon^3} \left(\frac{119070\sqrt{318}\zeta(3)^2}{7890481} + \frac{261252\sqrt{318}\zeta(3)}{7890481} \right. \\
& - \frac{4860\sqrt{318}\zeta(5)}{148877} + \frac{3\sqrt{83789895038142}}{63123848} \Big) - \epsilon^2 \frac{27}{836390986} \left(10668672\zeta(3)^3 \right. \\
& + 33819408\zeta(3)^2 + 2\zeta(3)(5195389 - 21636720\zeta(5)) + 3870802\zeta(4) \\
& \left. - 30191450\zeta(5) - 40879377\zeta(7) + 2892644 \right) . \tag{2.9}
\end{aligned}$$

The impure fixed point has been proved to be stable in the framework of the perturbation theory in $\sqrt{\epsilon}$. Hence, we have to insert u^* and v^* into the critical exponents series. The expansions for critical exponents in powers of $\sqrt{\epsilon}$ being the eventual goal of this study are as follows:

$$\begin{aligned}
\eta = & -\frac{\epsilon}{106} + \sqrt{\epsilon^3} \frac{9\sqrt{318}}{148877} \left(7\zeta(3) + 24 \right) - \epsilon^2 \frac{27}{63123848} \left(21168\zeta(3)^2 \right. \\
& + 76040\zeta(3) - 38160\zeta(5) + 22469 \Big) + \sqrt{\epsilon^5} \left(\frac{15752961\sqrt{318}\zeta(3)^3}{22164361129} + \frac{70314804\sqrt{318}\zeta(3)^2}{22164361129} \right. \\
& + 27\sqrt{318}\zeta(3) \frac{26313923 - 33657120\zeta(5)}{354629778064} + \frac{189\sqrt{318}\zeta(4)}{595508} \\
& \left. - \frac{4725\sqrt{467895342}\zeta(5)}{1672781972} - \frac{130977\sqrt{318}\zeta(7)}{63123848} + \frac{2997\sqrt{152112323838}}{44328722258} \right) , \tag{2.10}
\end{aligned}$$

$$\begin{aligned}
\nu = & \frac{1}{2} + \sqrt{\epsilon} \frac{\sqrt{318}}{212} + \epsilon \frac{535 - 756\zeta(3)}{22472} + \sqrt{\epsilon^3} \left(\frac{59535\sqrt{318}\zeta(3)^2}{31561924} + \frac{39555\sqrt{318}\zeta(3)}{15780962} \right. \\
& - \frac{1215\sqrt{318}\zeta(5)}{297754} + \frac{397\sqrt{705044478}}{504990784} \Big) - \epsilon^2 \frac{1}{6691127888} \left(288054144\zeta(3)^3 + 679447440\zeta(3)^2 \right. \\
& - 45\zeta(3)(25964064\zeta(5) + 8113195) + 168826518\zeta(4) \\
& \left. - 401571990\zeta(5) - 7(157677597\zeta(7) + 9164941) \right) . \tag{2.11}
\end{aligned}$$

The $\sqrt{\epsilon}$ and ϵ terms in Eq. (2.10) and first three terms in Eq. (2.11) coincide with those calculated earlier [15,16] while the rest are essentially new. Estimating coefficients in the above expansions numerically we obtain:

$$\eta = -0.0094339622\epsilon + 0.034943501\sqrt{\epsilon}^3 - 0.044864982\epsilon^2 + 0.021573216\sqrt{\epsilon}^5 \quad , \quad (2.12)$$

$$\nu = \frac{1}{2} + 0.084115823\sqrt{\epsilon} - 0.016632032\epsilon + 0.047753505\sqrt{\epsilon}^3 + 0.27258431\epsilon^2 \quad . \quad (2.13)$$

Corresponding $\sqrt{\epsilon}$ -expansion for the susceptibility exponent reads:

$$\gamma = 1 + 0.16823164\sqrt{\epsilon} - 0.028547082\epsilon + 0.078828812\sqrt{\epsilon}^3 + 0.56450485\epsilon^2 \quad . \quad (2.14)$$

The most striking feature of the series just obtained is the quite irregular behaviour of their coefficients. It may be considered as one of the main reasons of failure of our attempts to apply various resummation techniques to these expansions. Indeed, the numerical results found by means of several methods based on the Borel transformation turned out both to contradict to exact inequalities and to differ markedly from estimates given by 3D RG analysis [6,7,8] and by computer simulations [17]. For example, numerical estimates for the exponent ν thus obtained lead, via scaling relation $\alpha = 2 - D\nu$, to positive (and big!) values of the exponent α , in obvious contradiction with the inequality $\alpha < 0$ proven for impure systems [18].

The “odd” behaviour of coefficients of the $\sqrt{\epsilon}$ -expansions for critical exponents inherent in the RIM appears to be the rule rather than the exception, this apparently indicates on the Borel non-summability of perturbative series. The point is that the analysis of the perturbative expansions for the free energy of the zero-dimensional (“toy”) model with quenched disorder has lead to the conjecture that for disordered systems perturbative series are Borel non-summable [19]. Such an non-summability has then been related to Griffiths singularities [20]. So, the irregularity of signs and values of coefficients of the $\sqrt{\epsilon}$ -expansions found may be regarded as a manifestation of the Borel non-summability of these series.

III. IS A CUBIC CRYSTAL “ISOTROPIC” AT THE CRITICAL POINT?

Much better situation takes place in the case of a pure system with cubic anisotropy. Five-loop ϵ -expansion for the marginal spin dimensionality found by Kleinert and Schulte-Frohlinde [12]

$$\begin{aligned}
 n_c &= 4 - 2\epsilon + \epsilon^2 \left(-\frac{5}{12} + \frac{5\zeta(3)}{2} \right) + \epsilon^3 \left(-\frac{1}{72} + \frac{5\zeta(3)}{8} + \frac{15\zeta(4)}{8} - \frac{25\zeta(5)}{3} \right) \\
 &+ \epsilon^4 \left(-\frac{1}{384} + \frac{93\zeta(3)}{128} - \frac{229\zeta^2(3)}{144} + \frac{15\zeta(4)}{32} - \frac{3155\zeta(5)}{1728} - \frac{125\zeta(6)}{12} + \frac{11515\zeta(7)}{384} \right) \\
 &= 4 - 2\epsilon + 2.58847559\epsilon^2 - 5.87431189\epsilon^3 + 16.8270390\epsilon^4
 \end{aligned} \tag{3.1}$$

is seen to be alternating. Moreover, coefficients modulo of its Borel transform are easily shown to monotonically decrease what may be thought of as a manifestation of the Borel summability of the original series.

Let us calculate n_c for the three-dimensional system applying the Pade-Borel resummation technique to the expansion Eq. (3.1) and then putting ϵ equal to unity. In so doing, however, it is very important to trace how sensitive is the numerical estimate for n_c thus obtained to the perturbative order employed. That is why we calculate here n_c not only in the five-loop approximation but also in three- and four-loop orders in ϵ . Correspondingly, Pade approximants $[1/1]$, $[2/1]$ and $[3/1]$ are used for analytical continuation of the Borel transforms of the original series. The results obtained in three subsequent approximations mentioned are as follows:

$$n_c^{(3)} = 3.004, \quad n_c^{(4)} = 2.918, \quad n_c^{(5)} = 2.855. \tag{3.2}$$

These estimates are seen to behave quite regularly. They decrease with increasing the order of the approximation demonstrating the tendency to go deeper and deeper below 3. This fact is in agreement with the conclusion about the character of critical behaviour of cubic crystals made earlier on the base of higher-order RG calculations in three dimensions

[7,9,10]. Indeed, two-loop, three-loop [9,10] and four-loop [7] RG expansions for 3D cubic model resummed by means of the generalized Pade–Borel (Chisholm–Borel) method lead to the estimates:

$$n_c^{(2)} = 3.01, \quad n_c^{(3)} = 2.95, \quad n_c^{(4)} = 2.90. \quad (3.3)$$

These values being very close to their counterparts resulting from the ϵ -expansion Eq. (3.1) also go down when the order of the approximation grows up, and the most accurate 3D estimate available $n_c^{(4)} = 2.90$ is appreciably smaller than 3.

Pretty good agreement between the higher-order RG estimates for n_c obtained in three and $(4 - \epsilon)$ dimensions enable us to believe that both estimates are close enough to the exact value of n_c . Hence, we may conclude that $n_c < 3$ and cubic crystals with vector order parameter (3D spins) should demonstrate, in principle, anisotropic critical behaviour with critical exponents differing from those of the 3D Heisenberg model. On the other hand, since the difference between n_c and 3 is rather small the cubic fixed point of the RG equations should be located in close vicinity of the Heisenberg fixed point at the flow diagram. As a result, critical exponents describing the anisotropic critical behaviour should be numerically close to the critical exponents of the Heisenberg model.

IV. SUMMARY

In the paper, $\sqrt{\epsilon}$ -expansions for critical exponents of the weakly-disordered Ising model are calculated up to the five-loop order. Coefficients of the expansions obtained are found to exhibit rather irregular behaviour preventing these series from to be resummed by means of the procedures based on the Borel transformation. This fact may be thought of as a reflection of Borel non-summability of such expansions conjectured earlier on the base of studying of much simpler (zero-dimensional) model. The marginal spin dimensionality n_c

for a cubic model is calculated by the Pade–Borel resummation of corresponding five–loop ϵ –expansion obtained by Kleinert and Schulte–Frohlinde. The estimate $n_c = 2.855$ thus obtained is found to agree well with its analog extracted earlier from four–loop 3D RG expansions. The conclusion is made that the exact value of n_c is smaller than 3 and cubic crystals with vector order parameters should demonstrate, in principle, anisotropic critical behaviour.

ACKNOWLEDGMENTS

We are grateful to I. O. Maier who has attracted our attention to the paper [12]. One of the authors (BNS) has much benefitted from discussions with H. Kleinert and K. Ziegler. The work in Electrotechnical University was supported in part by the Foundation for Intellectual Collaboration (St. Petersburg) via Russian Scientific and Technological Program “Fullerenes and Atomic Clusters”, Project No. 94024.

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